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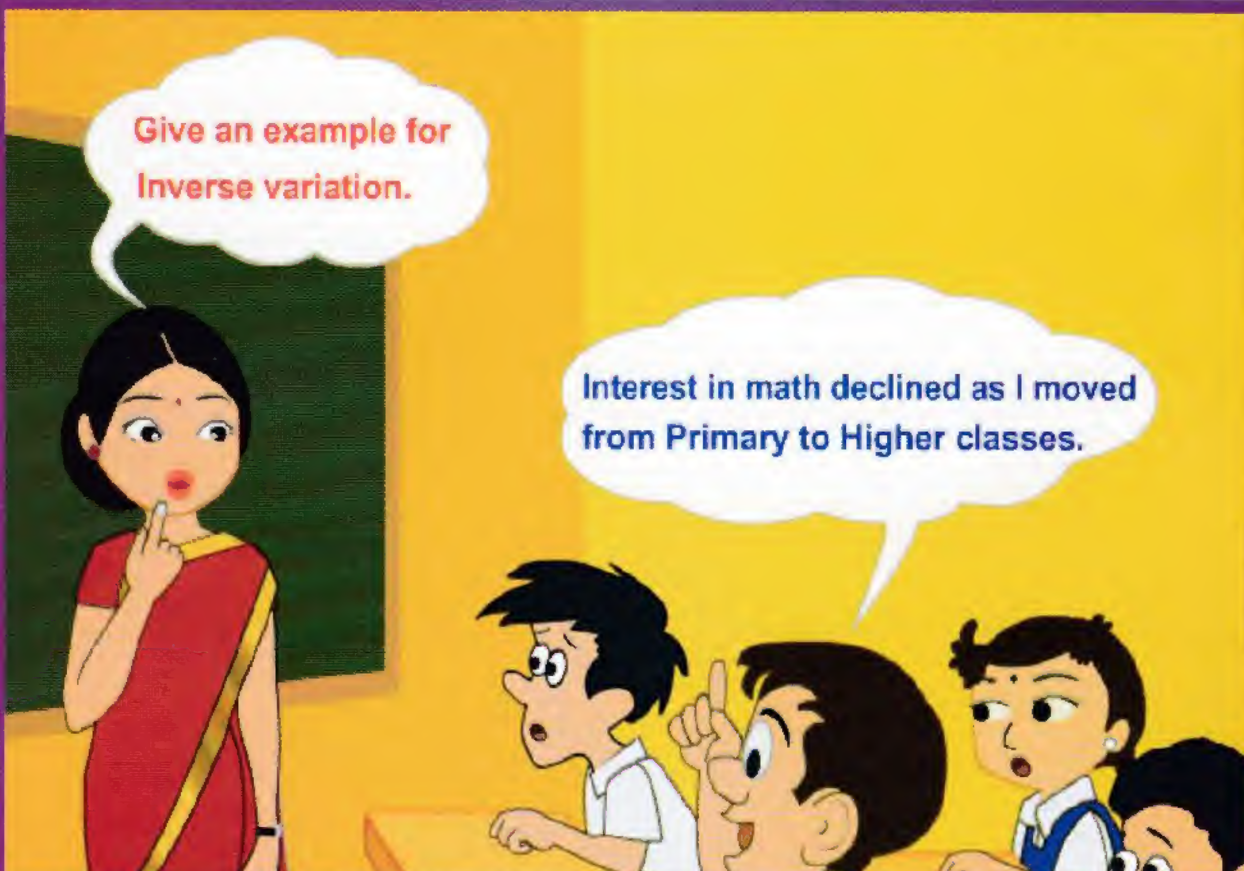

JM

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Junior mathematician (JM) is a Mathematics magazine, principally meant for youngsters of age between 8 and 16.

- Aims to interact directly with the fresh, young and receptive minds, motivating them in their appreciation and application of Mathematics.
- Aspires to present Mathematics as a lovable subject, satisfying to the serious-minded and pleasurable for others, removing math-phobia.
- Provokes young student-authors to write their own discoveries and creative thoughts.

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Names that inspire

We Indians take the credit for the invention of Zero as "a symbol, a number, a magnitude, a direction separator and a place-holder, all in one operating within a fully established positional number system". Apart from this, how many of us are aware of our own rich mathematical heritage or great Indian names, past and present, pertinently associated with enrichment of mathematics? Can you identify the people in the following photographs and say how their names glorify our heritage?

(See page 29 for a mini-outline on these personalities)

A look at some number patterns

S. Rangan& B. Jayasree (8^{th} std), Thirumalainagar, Chennai.

It is interesting to look at numbers and study their properties. Here we wish to give a few things that we found quite rewarding during such an exercise. (It was a 'vacation-task' given by our teacher.)

Let us take several types of numbers. The Blue numbers are our usual friends, namely the Natural numbers. The red numbers are odd numbers while the green ones are powers of 2. The brown numbers are squares whereas the pink ones are cubes. The final column is made of the prime numbers.

1	1	2	1	1	2
2	3	4	4	8	3
3	5	8	9	27	5
4	7	16	16	64	7
5	9	32	25	125	11
6	11	64	36	216	13
7	13	128	49	343	17
8	15	256	64	512	19
9	17	512	81	729	23
10	19	1024	100	1000	29
11	21	2048	121	1331	31
12	23	4096	144	1728	37
13	25	8192	169	2197	41
14	27	16384	196	2744	43
15	29	32768	225	3375	47
16	31	65536	256	4096	53
17	33	131072	289	4913	59
18	35	262144	324	5832	61
19	37	524288	361	6859	67
20	39	1048576	400	8000	71
21	41	2,097,152	441	9261	73
22	43	4,194,304	484	10648	79
23	45	8,388,607	529	12167	83
24	47	⋮	576	13824	89
25	49	⋮	625	15625	97
26	51	⋮	676	17576	101
27	53	⋮	729	19683	103
28	55	⋮	784	21952	107
29	57	⋮	841	24389	109
30	59	⋮	900	27000	113

We took several funny questions. *Suppose we take two numbers with the same colour, can we find a number of the same colour which will be the sum of the two numbers? This was ok with blue numbers but not in other columns. Same was true in their case of products. When we took subtraction none of these columns satisfied our conditions. For example we could not find 1 – 2 in Column 1 and 7 – 5 in column 6. (There were one or two occasions it happened but not always). Division failed in most cases in all the columns.

* *Ed* : Whenever such a condition on an operation (like addition, subtraction etc.) is satisfied in a collection, mathematicians say closure property is satisfied.

While checking for multiplication, we found a strange thing in some cases. For example, when you want to multiply two green numbers . You look at the corresponding blue numbers, add them and choose the green number that matches with this sum. For example, if you want **4** × **32**, you simply get **2** + **5** (**2** is against**4**and **5** is against**32**) and just look at the green number corresponding to **7**. You find **128**! We could not verify this for large numbers, since the products became larger and larger.(**Ed*: You can learn about this in Logarithms topic in higher classes.)

The red and brown columns gave a special relation. We found that **1**⇔1; **1**+**3**⇔4; **1**+**3**+**5**⇔9; **1**+**3**+**5**+**7**⇔16 etc. We are to add larger and larger red numbers like this to get the brown numbers. With some difficulty we understood that a square number can be written as a sum of consecutive odd numbers starting with 1.

We also found some squares of Pythagorean triples in the brown column: 9-16-25, 144-25-169, 576-49-625. We could not go quite farther; the squares were very large. We think corresponding to every odd square, we can get such results.

When we consider squaring the numbers, we found that if we square a brown number, we get another brown number. We understood that this natural because the square of a square must be a square. The pink numbers also behave similarly. If you square a cube number you get a cube number.

We noted that half of the blue numbers are red. We are not sure if this would be same when more and more numbers, blue and red, are taken.

The units digits in each column are interesting. The units digit of green numbers (**2, 4, 8, 6**) , of brown numbers (**1, 4, 9, 6, 5, 6, 9, 4, 1, 0**) and Pink numbers (**1, 8, 7, 4, 5, 6, 3, 2, 9, 0**) seem to repeat again and again cyclically.

Perhaps we can also look at the list of reciprocals of these numbers. We feel that many more wonderful patterns can be found if we continue to work at this project for a longer period. Our routine lessons do not permit such a desirable interval.

An attempt to relate the Central Angle and Total Angle of a polygon

SreedharK S, Class 9 , KendriyaVidhyalaya, Ramavarmapuram, Thrissur, Kerala

I tried to find a relation between the central angle and the total angle of a polygon. I share here my experience. My investigations led to the following result:

The sum of total angles of a polygon with n sides will be the product of central angles of a regular polygon with n-1 sides and the sum of numbers up to n-2. That is,

$$\overbrace{\frac{360}{n-1}}^{1} \times \overbrace{\frac{(n-2)+1}{2} \times (n-2)}^{2}$$

1) The central angle of the regular polygon with sides n-1.
   Central angle = $360 \div (n - 1)$
2) The sum of numbers up to $n - 2 = \Sigma(n - 2)$

$$= 1+2+3+4+\ldots+(n-1) = \frac{(n-1)\times n}{2}$$

Table

No. of sides	Total angle of polygon with n sides	Central angle of polygon with n-1 number of sides	Sum of numbers up to n-2
3	180°		
4	360°	120°	1
5	540°	90°	6
6	720°	72°	10
7	900°	60°	15
8	1080°	51.42857142857143°	21
9	1260°	45°	28
10	1440°	40°	36
11	1620°	36°	45
12	1800°	32.72727272727273°	55
13	1980°	30°	66
14	2160°	27.69230769230769°	78
15	2340°	25.71428571428571°	91
16	2520°	24°	105
17	2700°	22.5°	120
18	2980°	21.17647058823529°	136
19	3160°	20°	153
20	3340°	18.94736842105263°	171
21	3520°	18°	190

Here are some examples:

No. of sides of Polygon considered	Sum of its interior angles
7	$\frac{360}{6}\times\frac{(7-2)+1}{2}\times(7-2) = 60\times15 = 900$
10	$\frac{360}{9}\times\frac{(10-2)+1}{2}\times(10-2) = 40\times36 = 1440$
9	$\frac{360}{8}\times\frac{(9-2)+1}{2}\times(9-2) = 45\times28 = 1260$
16	$\frac{360}{15}\times\frac{(16-2)+1}{2}\times(16-2) = 24\times105 = 2520$
3	$\frac{360}{2}\times\frac{(3-2)+1}{2}\times(3-2) = 180\times1 = 180$
19	$\frac{360}{18}\times\frac{(19-2)+1}{2}\times(19-2) = 20\times153 = 3060$

Verification through number of examples helped me to ascertain that the formula conceived by me is correct.

A sudden flash and I found that my formula after all can be very much simplified as follows:

$$\frac{360}{n-1}\times\frac{(n-2)+1}{2}\times(n-2) = \frac{360}{(n-1)}\times\frac{(n-1)}{2}\times(n-2) = \frac{360}{2}\times(n-2) = (n-2)180$$

(*Ed.*: In search for a beautiful formula, the student had missed a simplified form of the result in the beginning! However, one should feel happy that his efforts were original and such things could happen in any exploration. Perhaps great mathematicians also underwent, in the past, similar shocks and surprises!)

Heard about M.C. Escher?

- *"I never got a pass mark in math ... Just imagine – mathematicians now use my prints to illustrate their books."*
  -- M.C. Escher

MauritsCornelis Escher was a Dutch graphic artist who made mathematically-inspired woodcuts, lithographs, and mezzotints.

Cross the bridge!

There are four people on one side of a bridge in the middle of the night. They only have one torch between them and all have to get across the bridge within 17 minutes.
However, only a maximum of two people may cross at one time and the torch must always be in the possession of one of the people crossing the bridge. All the people walk at different speeds, but any pair crossing must walk at the slower of the two speeds.

The people's speeds are as follows:
Person
Atakes 1 minute to cross.
B 2
C 5
D 10
How do you get all people across in that time frame?

How do you generalize?

Observe the following and generalize for a formula (most of you already know it!).

$$0 = \frac{1^2-1}{2} = \frac{1(1-1)}{2} = \frac{1\cdot 0}{2}$$

$$0+1 = \frac{2^2-2}{2} = \frac{2(2-1)}{2} = \frac{2\cdot 1}{2}$$

$$0+1+2 = \frac{3^2-3}{2} = \frac{3(3-1)}{2} = \frac{3\cdot 2}{2}$$

$$0+1+2+3 = \frac{4^2-4}{2} = \frac{4(4-1)}{2} = \frac{4\cdot 3}{2}$$

$$0+1+2+3+4 = \frac{5^2-5}{2} = \frac{5(5-1)}{2} = \frac{5\cdot 4}{2}$$

$$0+1+2+3+4+5 = \frac{6^2-6}{2} = \frac{6(6-1)}{2} = \frac{6\cdot 5}{2}$$

$$0+1+2+3+4+5+6 = \frac{7^2-7}{2} = \frac{7(7-1)}{2} = \frac{7\cdot 6}{2}$$

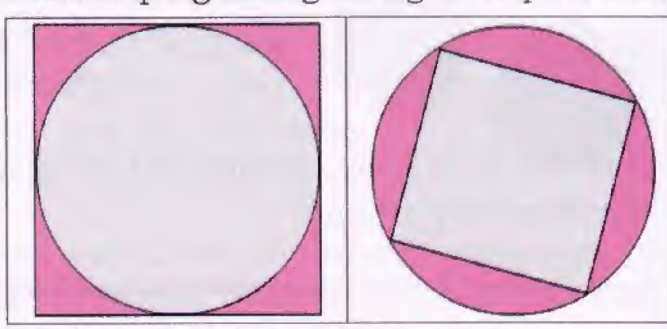
(Go to last page for answers)

A square peg in a round hole

-V. Sundaramurthy, Teacher Educator, Chennai.

Have you ever heard the phrase "fitting like a square peg into a round hole"? It's meant to suggest an obvious mismatch between a person, thing, or idea and some situation. Which is a better fit, a round peg in a *square hole*, or a square peg in a *round hole*? That would be interesting for a student of mathematics.

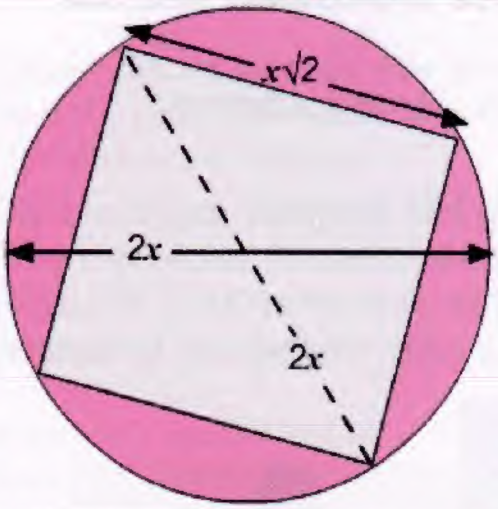
We now explain this problem with two pictures that you see here.. On the left we have a square hole with a round plug fitting as tight as possible. On the right, we have the opposite; a round hole with a tightly fitting square plug.



Which is a *better fit*? We must be clear what we mean by "better fit". It is the design that blocks out the highest percentage of the original opening.

In the two pictures shown, you can see the gaps exhibited in **pink**. Our solution must have the *least* percentage of **pink** as a ratio of the original hole.

Square Peg in a Round hole

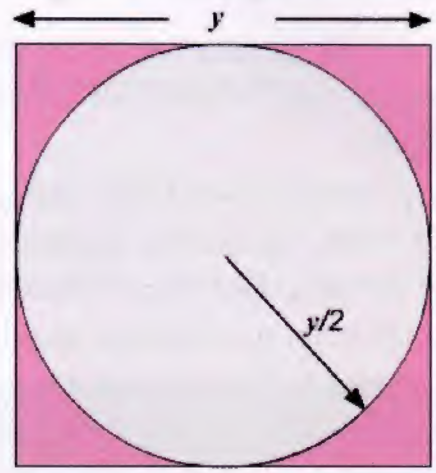


Let x be the radius of the round hole. Then the area of the round hole is πx^2. A diagonal of the square is same as a diameter of the circle.

$\therefore$ A side of the square = $\frac{2x}{\sqrt{2}} = x\sqrt{2}$

$\therefore$ The area of the square is = $2x^2$. The proportion of the area taken by the square peg in the round hole = $\frac{2x^2}{\pi x^2} = \frac{2}{\pi}$ ....(A)

Round Peg in a Square hole



Let y be the length of the side of the square here. Then the area of the square is y^2. A tightly fitting round plug in this hole has a radius = $\frac{y}{2}$.

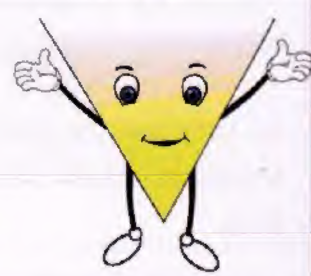
The area of this circle is $\pi \times \left(\frac{y}{2}\right)^2$.

The proportion of the area taken by a round peg in a square hole is = $\frac{\pi y^2 / 4}{y^2} = \frac{\pi}{4}$ ....(B)

Now A − B = $\frac{2}{\pi} - \frac{\pi}{4} = \frac{8-\pi^2}{4\pi} < 0$, since $\pi > 3$. This means A < B.

It seems to be more beneficial to put a round peg in a square hole than the other way around!

The best angle!



The only angle from which to approach a problem is the TRY-Angle !

Bingo!

T. Srinivasamurthy, (Home Schooling), Thillai Nagar, Salem.

Bingo is a game in which each player has a card with numbers on. Someone calls out numbers at random. If you are the first person to have all your numbers called out, you win the game. And you shout "Bingo" in excitement!

Under the direction of our teacher, we played Bingo, as described in a journal which she had brought to the class. I am sharing the excitement with JMs.

Each one of us was asked to prepare a Bingo sheet, a sheet of paper in which a 3×3 table is drawn; the funny part is that you are *freely allowed*to fill up the nine empty squares with *your own* choice of single digit numbers. Five 5 such sheets of students, whom I would call for simplicity, A, B, C, D and E, are shown hereunder.

A

12	12	12
12	12	12
12	12	12

B

5	3	4
11	7	10
8	0	9

C

9	7	1
5	4	8
16	7	8

D

7	8	6
9	7	5
8	5	9

E

13	11	10
12	9	8
7	6	14

We were told that the teacher would call numbers and each one of us should cross out the numbers, if found in their sheets. The first person to cross out all nine entries will be the winner. Our teacher used a pair of dice to call out the numbers; she simply added the numbers shown by the dice.

When all the nine entries had been called out, the winner was player D. What gave him the advantage in the game? We were curious to know. We examined D's sheet and compared it with the sheets of many other students. Most of us agreed that the cards of A and E were very bad. When you roll a die, you get numbers 1,2,3,4,5 or 6. Thus A and E had poor chances of winning, we felt. There were some 'impossible' numbers in the sheet of C and B. D had a fair chance of winning because he has

chosen numbers from the collection of possible outcomes when two dice are rolled.

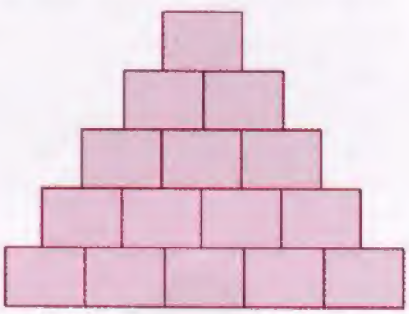
As a next game, the conditions were slightly changed. Instead of winner being the one who crosses out all the entries, it was announced that victory will be to the student who crosses digits in a line (one of diagonal, vertical or horizontal). The class ended with arguing if the way the digits are placed in the array has any role to lead to the win.

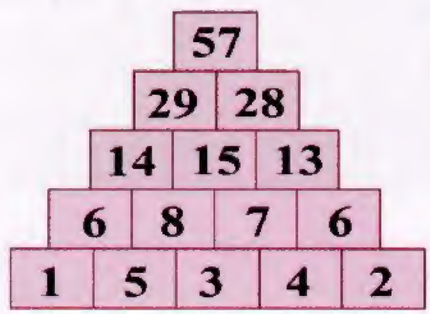
Quite exciting, of course!

(*Ed.* Many Bingo games are available on the Web. You have varieties for almost all topics in school mathematics. JMs may spend their spare time usefully playing such games.

Here is another well-known game, The Number Tower Game that can be made more challenging with additional conditions.

The game involves placing numbers, say, 1,2, 3, 4, and 5 at the base of the tower and build it as follows: Add two adjacent number boxes and write the total in the number box that stands above them.





A blank Tower   *A filled up Tower.*

Whatdifferent totals can you make using the bottom row? Since you are free to alter the order in the row, which order will turn out the greatest number at the top? Is it possible to find a method of arriving at such maximum value? If instead of taking consecutive natural numbers, if we take digits at random, how does the game change? What if we use fractions or decimal numbers? Suppose if the 'addition game' is changed into a 'multiplication game', how do the answers for the above questions vary?

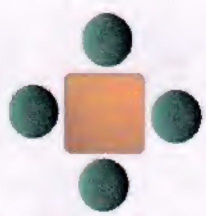
Yes, JMs, you can be a creator of more such investigations! Carry on!

Table and chair Problem

B. Sujata, West Veli St., Madurai.

This problem arose when we were arranging tables and chairs for the final year students' farewell function and tea-party. We from 9th standard were hosts and the preparations for the gala occasion were entrusted to us.

The predicament started when we started setting up the seating pattern for participants. It was our math teacher who started provoking us. The tables were all square-shaped.



Consider one table.
Four people can sit together at one table.

If two tables are placed together, side by side, then six people cansit together.

Several questions were raised by many students. What will happen if three tables are put side by side? Four tables? Five tables? Is there any pattern that arises out of such adding tables one after another? The outgoing finalists and the hosts (9th standard) in total are 125 students. In that case how many tables would be required for the arrangement?

There seemed to be a pattern.

$1 \mapsto 2\times1 + 2 = 4$

$2 \mapsto 2\times2 + 2 = 6$

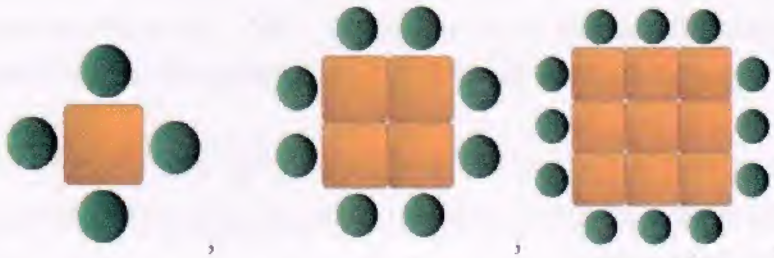
$3 \mapsto 2\times3 + 2 = 8$ ...... so on

No. of tables	1	2	3	4	...
No. of seats	4	6	8	...	...

After discussion, we found that any entry in the first row of the table when doubled and added to 2 gives the corresponding entry in the second row. With the help of our teacher we understood that this is an algebraic relation given by $y = 2x + 2$. Using this we could solve many queries that

arose earlier. But there were newer problems now, getting ready for discussion.

Why should one put the tables side by side? Why can't we arrange them in a different pattern as follows?



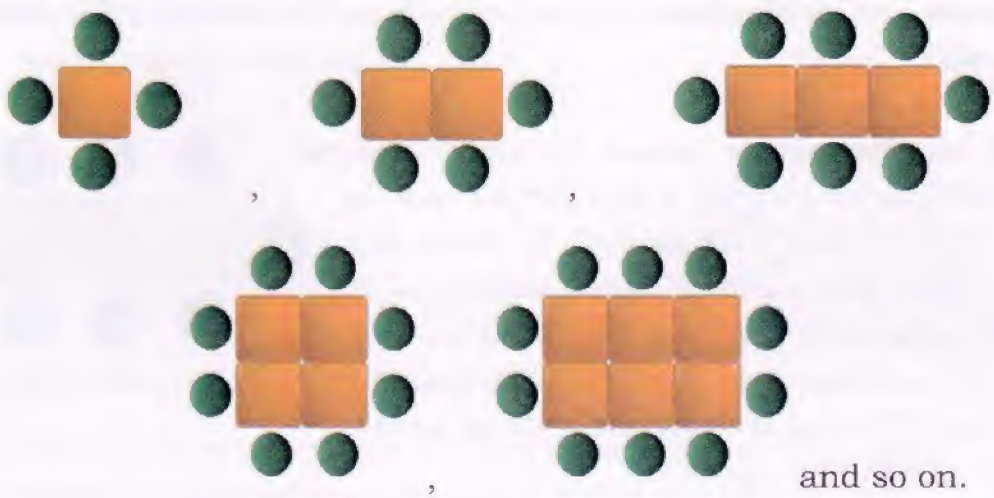
, and so on.

Now the table we prepared was different.

This gave us the rule $y = 4x$.

No. of tables	1	2	3	4	...
No. of seats	4	8	12	...	...

Some of us thought of tables getting fitted together in rectangles or squares of successively larger areas. The scheme was as follows:



and so on.

When tabulated, the relation between the tables and seats was given by the table:

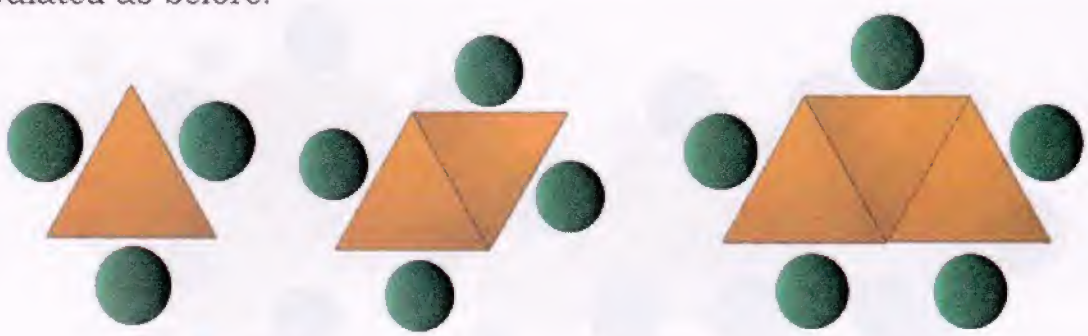
No. of tables	1	2	3	4	6	...
No. of seats	4	6	8	8	10	...

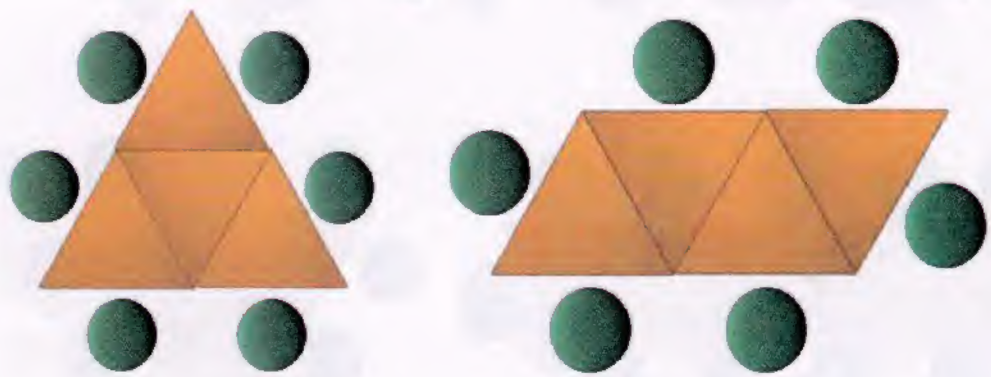
In this pattern we noticed a funny thing. When the number of tables increased from 3 to 4, there was no advantage, since the number of seats remained unchanged. We understood that an increase in the number of tables need not always lead to an increase in the number of tables. We tried to extend the table

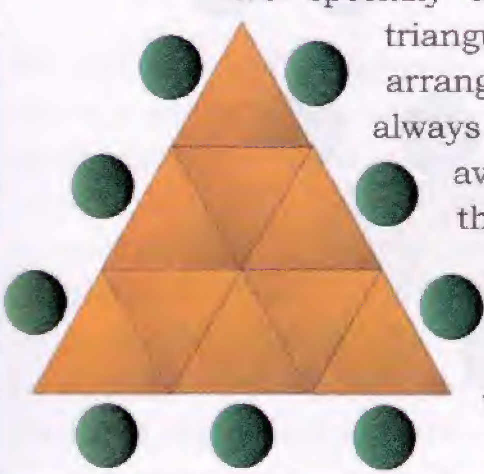
further by drawing pictures, but found it difficult to coin an algebraic rule connecting the number of tables and number of seats.

Though we had different options as above, the space available for arranging the get-together (its extent, shape and ventilation) had also to be considered and we decided to take the last option as a feasible one.

However, some of us continued to discuss about various possibilities in a funny way. What will happen if the tables were not square-shaped but of shapes of equilateral triangles? We drew some rough figures and then tabulated as before:



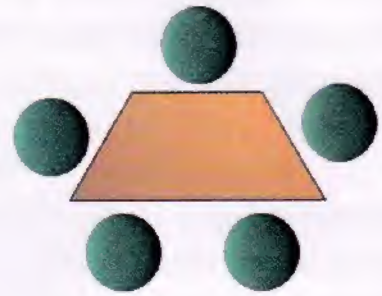


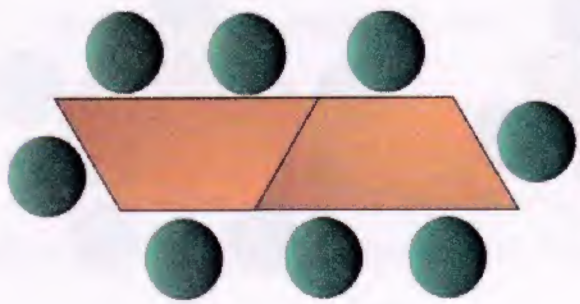


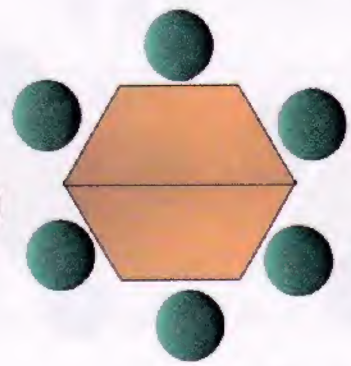
We specially noted that in these arrays of equilateral triangular tables, four tables could easily be arranged in two different ways. Such a possibility always helps to arrange tables to suit the availability of space. When in such parties, the idea of aesthetics also is given importance, the preference for arrangements do not entirely depend on mathematical notions only. If we are hiring tables, we have to take into consideration the cost factor also.

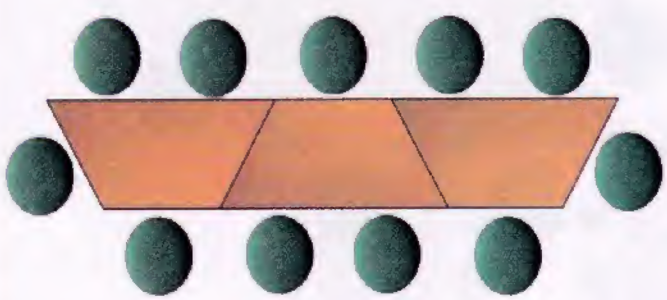
Next, we ventured to figure out the number of seats in the case of regular hexagonal tables.

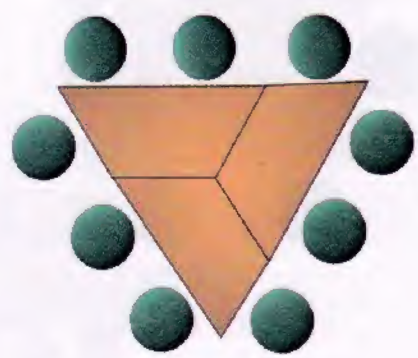
Here we list the table shapes formed by usage of identical tables in the shape of a trapezium. Several varieties are possible.

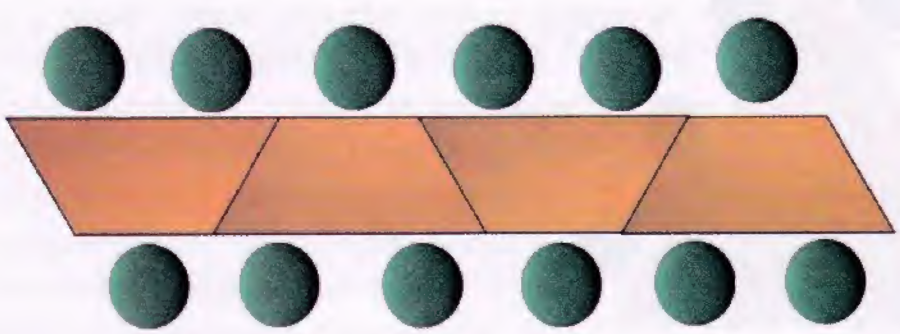




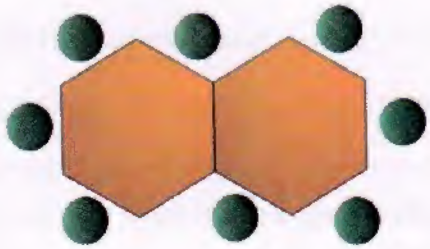
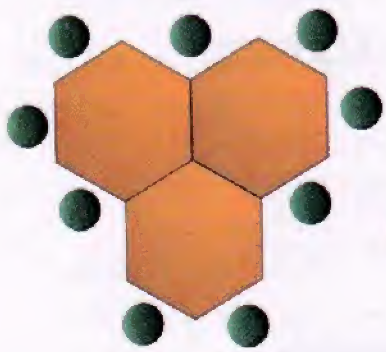
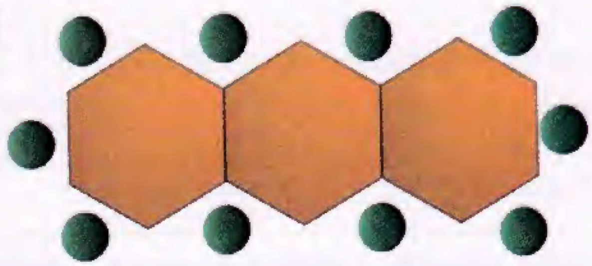


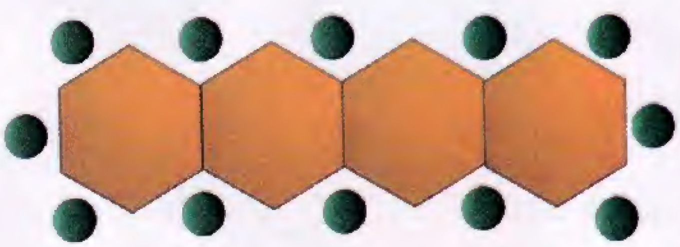






How about hexagonal table arrangement? Here are some samples.



We can thus use geometrical seating arrangements, compute the number of seats which can be accommodated, prepare a table and perhaps in most cases should be able to find an algebraic formula relating the number of tables and the number of seats. Such an approach helps in general making seating arrangements compact and tidy. Perhaps with higher math concepts, it should be possible to include other constraints such as space, number of people available for service etc. This may not be necessary in small arrangements for get together but will certainly be useful in bigger and complicated table-seat arrangements.

A fallacy!

When $n = -1\frac{1}{2}$,

$$(n + 1)^2 = n^2 + 2n + 1 = (-1\tfrac{1}{2})^2 + 2(-1\tfrac{1}{2}) + 1 = 2\tfrac{1}{4} - 3 + 1 = \tfrac{1}{4}$$

$$(n + 2)^2 = n^2 + 4n + 4 = (-1\tfrac{1}{2})^2 + 4(-1\tfrac{1}{2}) + 4 = 2\tfrac{1}{4} - 6 + 4 = \tfrac{1}{4}$$

Therefore, $(n+1)^2 = (n+2)^2$ ....(1)

Taking square root on both sides $n+1 = n+2$

which reduces to $1 = 2$

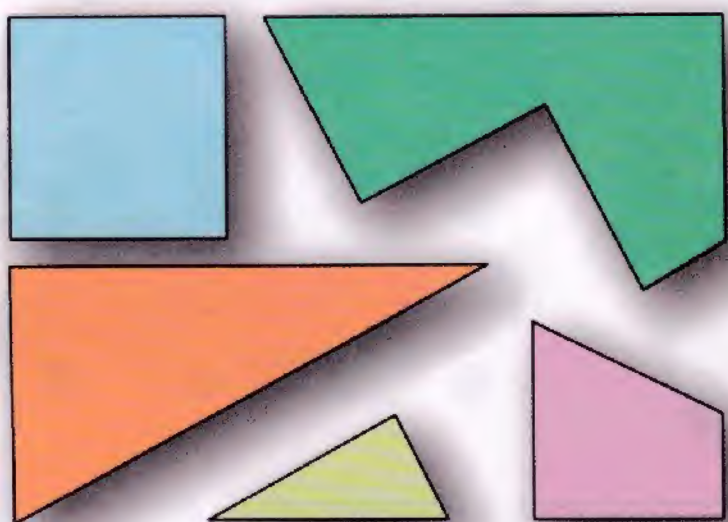
Can you find the fallacy in the argument?

Make a Geometric figure

The following five pieces were cut off from a Square shape. Take a copy of these five cut-outs and arrange them to form a single large

(1) Square (2) Rectangle (3) Triangle

(4) Parallelogram (5) Rhombus (6) A Swiss Cross.



(Go to last page for answers)

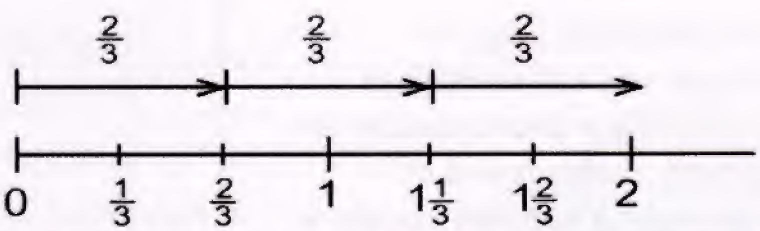
Think about it!

Three students were given a problem. After approximating the solution, they got the answers 8, 8.0 and 8.00. Do these answers have different meanings?

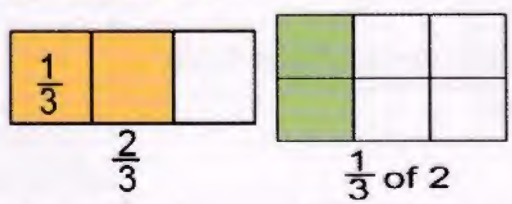
Equivalent fractions

S. Vinay Kumar, Venus Turorial Institute, Anantapur.

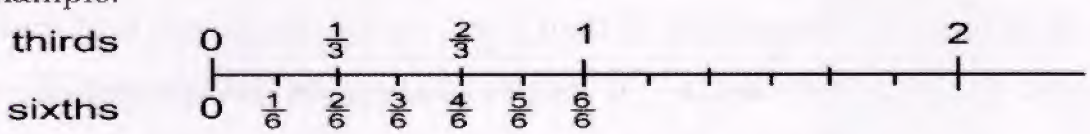
We had studied fractions in our lower classes. When we started learning fractions, we came across many difficult things to grasp the concepts. For example, we were told that ⅓ is same as 1 divided by 3. When $\frac{2}{3}$ is taken up, we interpret it as 2 times $\frac{1}{3}$ and also as 2 divided by 3. So, should we consider it as one-third of 3 or two one-thirds? Does this depend on the context? If we use a number line the whole thing seems to be complicated.



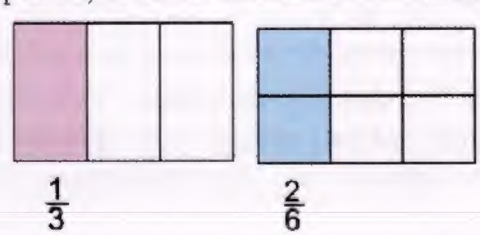
However, the coloured parts in the following diagrams seem to be simple.



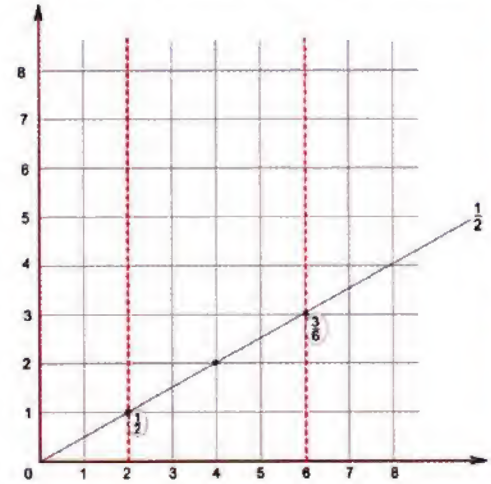
The idea of equivalent fractions has another strange thing: we use different fractions and then say they are all same and *represent the same number*. If you consider number line, then it may be like this, for example:



Or, in terms of shaded parts, it is illustrated as follows.

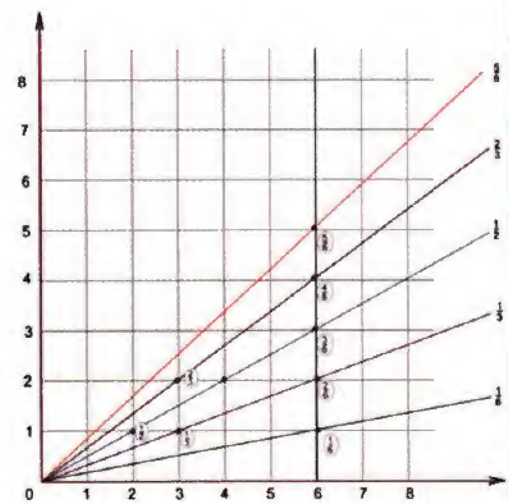


When we were discussing all these in our class, our teacher introduced to us one more way of symbolizing equivalent fractions, based on the ideas of coordinates which we have studied. Here, equivalent fractions are shown by *lattice points. These points have positive coordinates that lie on a line through the origin, the slope of the line being in each case the corresponding rational number.



For example if the slope of the line is ½, it passes through the lattice points (2,1), (4,2), (6,3), (8,4), ... These demonstrate that $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\ldots$

This sort of visualization helps to illustrate addition and subtraction of fractions nicely.Using a figure similar to the one given here, one can add or subtract fractions by a suitable choice of vertical line as the number line.



For e.g., look at the blue vertical line from which you can see that

$$\frac{2}{3}-\frac{1}{2}=\frac{4}{6}-\frac{3}{6}=\frac{1}{6}$$

(*Note: A lattice point is the intersection of two or more grid lines).

You can also use this type of diagram to compare fractions. This can be done by comparing their slopes of their lines. In the figure you find that the green line (that represents $\frac{2}{3}$) is less steep than the red line (which stands for $\frac{4}{5}$) and thus $\frac{2}{3}<\frac{4}{5}$.

(*Ed.* Here are a few things for related investigation. You talked about lines with rational slopes. Do lines with irrational slope intersect lattice points? A line with an irrational slope intersects at most one lattice point. What will happen if it intersected two lattice points?)

The Ladder and Box Problem

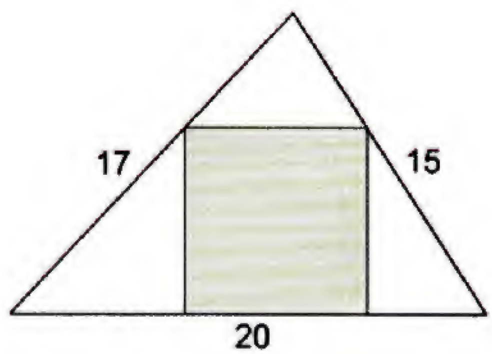
S. Ramesh & K. Karthik, Nanganallur, Chennai

What follows is a couple of problems which we discussed in one of our leisure periods in our class. It all started when the teacher who had come as a substitute for our math teacher (who was absent) challenged us if we could solve some tricky problems.

The first problem was about a square and a triangle:

A triangle has sides of lengths 20, 15 and 7. A square is inscribed in the triangle. The other two vertices of the square touch the two shorter sides of the triangle. What is the length of the side of the square?

We drew a rough figure.



Since the three sides of the triangle were given, its area could be calculated easily, using Heron"s formula:

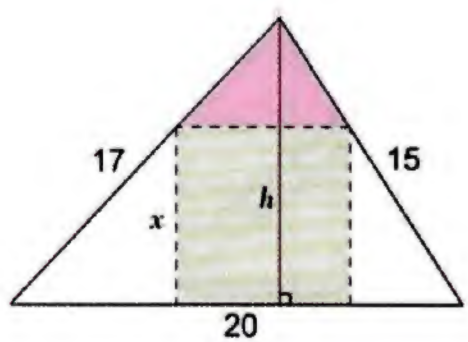
Area = $\sqrt{s(s-a)(s-b)(s-c)}$

where $s = semi - perimeter \ \dfrac{a+b+c}{2}$

Here $s = \dfrac{20+15+7}{2} = \dfrac{42}{2} = 21$ and so

area $\sqrt{21(21-20)(21-15)(21-7)}$

$= \sqrt{21\times1\times6\times14} = \sqrt{1764} = 42$



If h is the altitude drawn on side of length 20, then we must have

½ × base × altitude = 42.

i.e., $\frac{1}{2} \times 20 \times h = 42$

So, we get $h = 4.2$

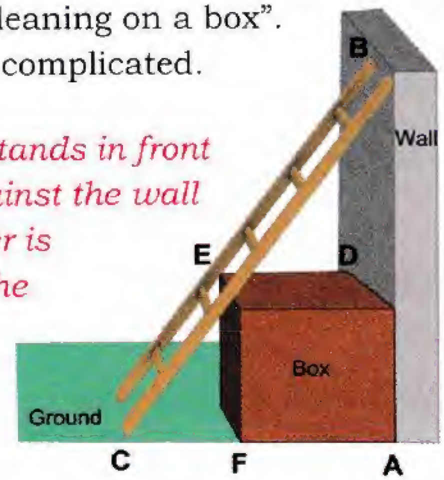
In the figure given, let x denote the side of the square. Note also that the pink triangle is similar to the whole given triangle.

Compare the correspondingratios of the similar triangles. It is easy to see

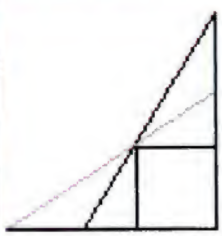
$$\frac{4.2}{20}=\frac{4.2-x}{x}\text{ giving } x=\frac{420}{121}=3.47\,(nearly).$$

Therefore, the length of the side of the square is 3.47 approximately.

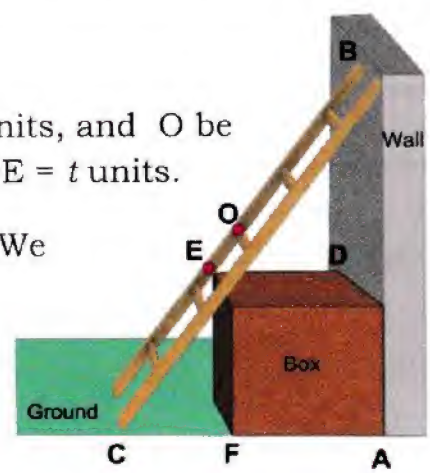
The real challenge was the next problem (we were told later that it is a famous classic problem) "problem of a ladder leaning on a box". It was a similar looking problem, a little more complicated.



A box in cube shape with the edge length 3m stands in front of a wall. A ladder of the length 10m leans against the wall and just touches the box at an edge. The ladder is divided into two unequal section bounded by the box to the ground and the box to the wall. what are those dimensions?



Note: More than one way of visualizing the situation is possible. We will consider the ladder in the steeper position, since we quite often would be interested in maximal height.



In the diagram, let BD = x units and CF = y units, and O be the midpoint of the length of the ladder. Let OE = t units.

Consider the similar triangles BDE and EFC. We have $\frac{BE}{EC}=\frac{BD}{EF}$ and so $\frac{5+t}{5-t}=\frac{x}{3}$ ....(1)

Also, from the right angled triangle BDE,

$$(5+t)^2 = x^2 + 9 \quad ....(2)$$

From (1) we get $x=\frac{3(5+t)}{(5-t)}$. Substituting this in (2), we get

$$(5+t)^2 = \frac{9(5+t)^2}{(5-t)^2}+9$$

Or $$(5+t)^2(5-t)^2 = 9(5+t)^2 + 9(5-t)^2$$

$$(5+t)^2(5-t)^2 = 9(5+t)^2 + 9(5-t)^2$$

$$(25-t^2)^2 = 9[(5+t)^2+(5-t)^2]$$

$$625-50t^2+t^4 = 9[50+t^2]$$

simplifies to $\quad t^4-68t^2+175=0$.

This is a fourth degree equation. To solve it, let us put $u = t^2$. We get

$$u^2-68u+175=0$$

$$t^4-68t^2+175=0$$

$$u^2-68u+175=0$$

Solving

$$u = \frac{68 \pm \sqrt{4624-700}}{2}$$

$$= 34 \pm \sqrt{981} \quad = 34 \pm 31.3209$$

$$= 34 \pm 31.3 \textit{ nearly.}$$

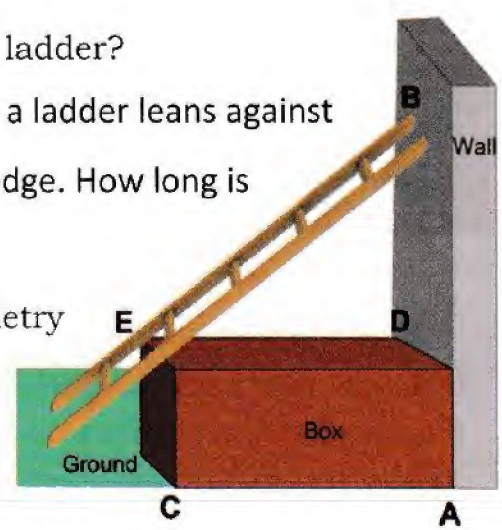
$$= 65.3 \textit{ or } 2.7$$

This gives $\quad t = \sqrt{65.3} \textit{ or } \sqrt{2.7} \; = 8.08 \quad \textit{or} \quad 1.64$.

It is now possible to compute the required results.

There were some additional questions for further discussion. Spme of them are:

1. How high on the wall is the top of the ladder?
2. The box is as seen earlier. Now suppose a ladder leans against the wall and just touches the box at an edge. How long is the shortest ladder?
3. Can you use ideas of coordinate geometry or trigonometry to solve some of these problems?
4. What if the cubic box is replaced by box in the shape of a cuboid? (See figure given).



Fraction Projects

Here is a list of activities which our class recorded while preparing for the math day exhibition, held in a small manner. Perhaps this could be useful to JMs. (We could complete only a few of them).

01. Try to explain visually the following situations involving fractions:

a) A fraction is a part of a Whole (Using a collection of discrete objects as well as with continuous situations like area, volume)

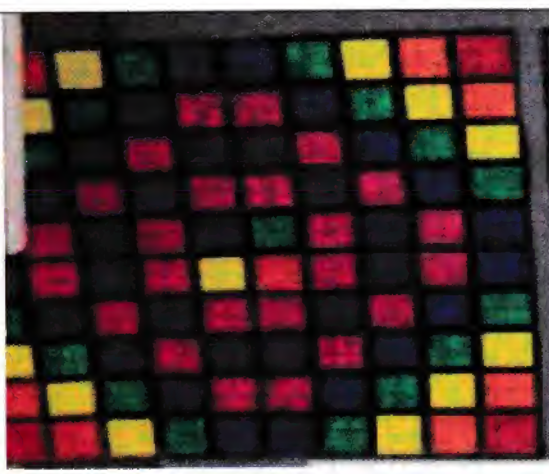
b) Two-thirds of 12 balls is two copies of one third of 6 balls.

c) $\frac{11}{4}$ is same as 11 divided by 4.

d) $\frac{2}{7}+\frac{4}{7}=\frac{6}{7}$ *e)* $\frac{2}{3}$ *of* $\frac{4}{5}=\frac{8}{15}$ *f)* $\frac{1}{(1/2)}=2$

g) $\frac{4}{4}=1$ *h)* $\frac{4}{1}=4$ *i)* $\frac{2}{3}\div\frac{3}{2}=1$

j) $\frac{6}{11}<\frac{5}{9}$ *k)* $3\frac{1}{4}\times 2\frac{2}{3}=8\frac{2}{3}$ *l)* $\frac{8}{10}\div\frac{2}{5}=2$



02. Use pattern blocks (or by other ways) to make beautiful designs. Indicate the fraction represented by each different group of blocks. (For example what fraction is, the space in the whole design, occupied by the 'trapezium' blocks?).

03. Prepare a short paper on Egyptian Fractions. Explain what they are and how they work. Then compare the Egyptian Fraction method with

the method we use today. What are the advantages and disadvantages of Egyptian Fractions? (A good reference is available at the site http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/egyptian.html.)

04. Here is an interesting way to connect the idea of fractions with decimals, percentages, ratios, equivalence, simplification etc. Greater the number of petals more will be points to be awarded for the exhibit.

05. Go through popular dailies of the month; identify contexts where ideas related to fractions are used while conveying the news. (For example: 'a third of the Budget allotted to Salaries', 'accident averted in a fraction of a second', 'two-thirds of this villagers do not have toilets' etc.). Prepare an album of these 'newspaper cuttings' and interpret the data mathematically.

06. Are there stories, fiction, dramas or movies where the concept of a fraction has a role to play? (There may be a movie on partition of property, the story of a monkey as a mediator during the wrangle between cats in dividing a loaf of bread into fractional parts, or a fiction describing how a detective works on fractional parts of a broken material etc). They may be collected and displayed.

07. A plan to improve your locality by converting a piece of waste land into a park where you allot different fractional areas for planting various plants and other purposes, justifying the choice of the allotments you make in the layout. A plan, the partitions and the cost could be exhibited.

08. Prepare a few games based on fundamental Operations on fractions (like addition, subtraction, etc) and exhibit them. As an example here is an incomplete magic square. To complete it you require basic skills of addition and subtraction among fractions. (Try it!).

		2
	$\frac{2}{3}$	
$1\frac{2}{3}$		$\frac{2}{3}$

Or challenges like: "How do you make the greatest mixed number using three numbers, when three dice are thrown simultaneously?

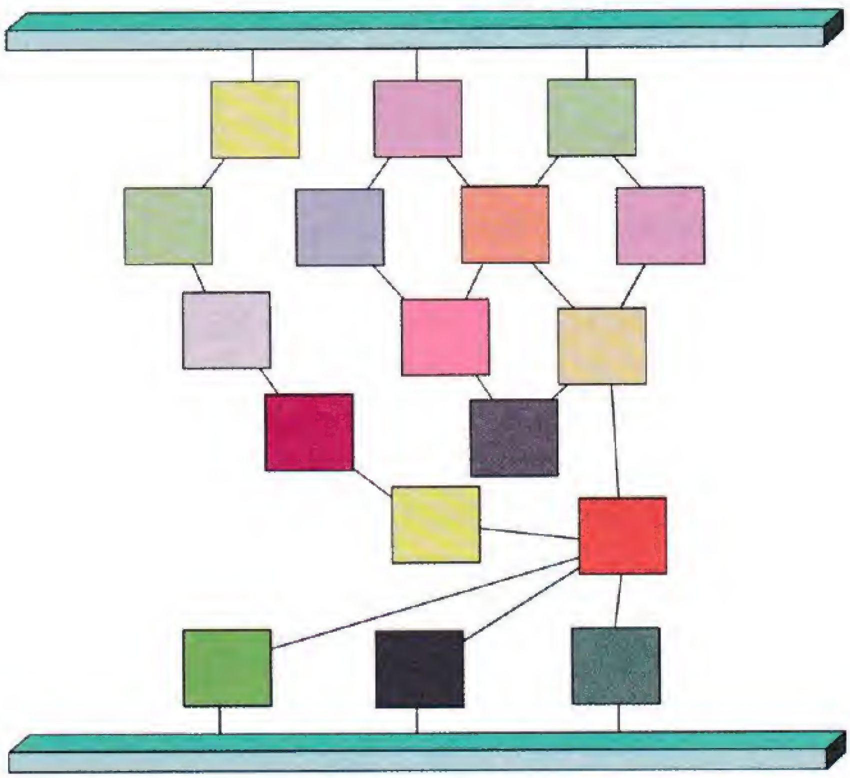
For example, (i) If the numbers seen are 2, 3 and 5, it can be 6½. (Can we say $5\frac{3}{2}$ is same as 6½?). (ii) When you arrange the digits 1,3,5,6 into two fractions, what will be the greatest sum?

A Game on Number Operations

S. Rangan & B. Jayasree (8^{th}std), Thirumalainagar, Chennai.

In the following is described a simple game in which mastery of number operations play a major role.

Two pupils, say A and B, can play this game. Each player needs a copy of the work-sheet shown here. They also need three dice.



Write the numbers 1 to 17 in the boxes, in an arbitrary manner. The players take turns to throw the three dice. The numbers on the dice are to be mingled with the use of mathematical operations to generate a result that should fall within the range 1-17. The resulting number is crossed off on the player's sheet. (Only one number has to be crossed out each turn). For example if player A throws 3, 4 and 8, she can delete numbers such as

15 (=3+4+8) or **7** (8+3–4) or **4** (=3×4–8) or **1** (= $3-\frac{8}{4}$) etc.

The winner is the first player to delete a complete chain of boxes linking the top of the chart to the bottom.

Extension: Instead of the first 17 natural numbers, use the first 17 prime numbers. It is more challenging!

Investigate Isosceles Numbers

Consider the sequence: 1, 3, 7, 13, 21, 31, ... (1)

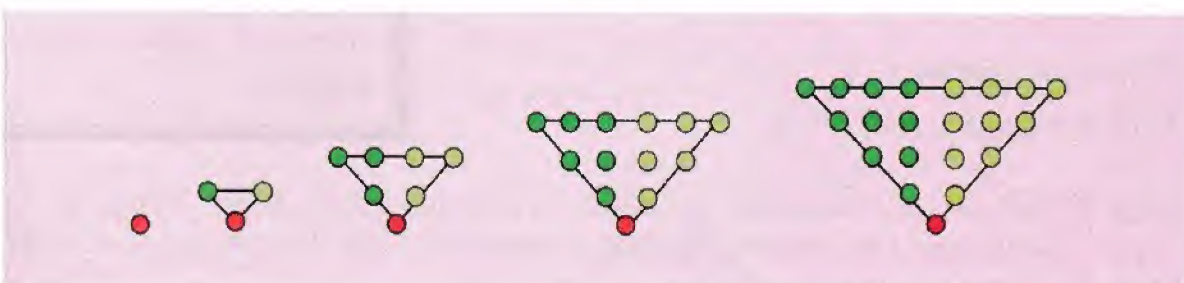
Do you see how the terms are found one by one in order?

Term		
1^{st}	1	1
2^{nd}	1+2	1+2(1)
3^{rd}	1+2+4	1+2(1+2)
4^{th}	1+2+4+6	1+2(1+2+3)
5^{th}	1+2+4+6+8	1+2(1+2+3+4)
6^{th}	1+2+4+6+8+10	1+2(1+2+3+4+5)
		$\vdots$
n^{th}	$1+2+4+6+\ldots+2(n-1)$	$1+2[1+2+3+\ldots+(n-1)]$

Let us try to give a visual representation to this:

Since the visuals look like Isosceles triangles, let us call these numbers given by the sequence (1) as isosceles numbers. If you want to have a formula for the n^{th} term (denote by t_n) of this sequence, you have

$$1+2[1+2+3+\ldots+(n-1)] = 1+2\left[\frac{n(n-1)}{2}\right] = 1+(n^2-n) = n^2-n+1$$



a) Try to show that $t_n = 2(n-1)+t_{n-1}$.

b) Why is every term odd here?

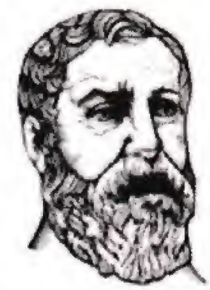
c) Is there an isosceles number that is also a square number?

d) Have you heard about triangular numbers? Are there isosceles numbers that are also triangular?

Heron's Method to find Square Root

M. Adaikkalanathan, Harur.

We have learnt about Heron's formula for the area of a triangle when the lengths of its three sides (say) *a*, *b* and *c* are given, namely $A = \sqrt{s(s-a)(s-b)(s-c)}$, s being the semi-perimeter $(\frac{1}{2})(a+b+c)$ of the triangle. We were surprised during a lecture (delivered on National Mathematics Day) to hear Heron's name in connection with a method of finding an approximate square root for positive integers.



Heron, (also known as Hero) was a Greek mathe- matician. When he lived is not well established. It may be around 0 AD. One of his most famous books, Metrica, consists of three volumes, and shows ways to calculate area and volume, and their divisions.

The method is as follows:

Step 1: Consider an approximate square root, say *s* of the given number *n*.

Step 2: Calculate the average of *s*and $\frac{n}{s}$.

Call this s_1.

Step 3: Find s_2 using s_1, similarly.

Repeat this process again and again, till you get the desired accuracy.

Here is an example:

Find the square root of 13.

Step 1: Let us take the root to be 4 as a first approximation. (This is because, the closest square number to 13 is 16, whose root is 4)

Step 2: The next approximation s_1 is given by

$$s_1 = \frac{n+s}{2} = \frac{13}{4} = 3.25$$

$3.25 \times 3.25 = 10.5625$ which is not a good approximation to 13

Step 3: The next approximation

$$s_2 = \frac{n+s_1}{2} = \frac{4+3.25}{2} = 3.625$$

$3.625 \times 3.625 = 13.140625$ which is closer to13

$\therefore$We can say $\sqrt{13}$ is nearly 3.625.

If we want, we can go further for still better approximation.

Quadratic formula

S. Guruprasad, 'Sabarinivas', Pappanaickenpalayam, Coimbatore.

I learnt from my uncle an unusual way of proving the formula for solving the quadratic equation $ax^2+bx+c=0\,(a,b,c \text{ are real numbers}, a\neq 0)$. (It seems he had read about it in some journal).

Given $\quad ax^2+bx+c=0\ \ (a,b,c \text{ are real numbers}, a\neq 0)$. ....(1)

Whatever may the value of x, you can write $x = (x-d) + d$.(Note that d can be any real number).

Let $t = x - d$, so that $x = t + d$..(2)

Substituting (2) in (1), we get $a(t+d)^2 + b(t+d) + c = 0$.

Therefore, $at^2 + (2ad + b)t + ad^2 + bd + c = 0$.

Remembering that d can have *any* desired real value, choose d such that the coefficient of t is zero.This means $2ad + b = 0$; and hence $d = -\dfrac{b}{2a}$, terminating the t term.Thus finally we get

$$at^2 + a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c = 0$$

$$at^2 + \frac{b^2}{4a} - \frac{b^2}{2a} + c = 0$$

$$at^2 - \frac{b^2}{4a} \qquad + c = 0$$

$$\Rightarrow \qquad at^2 = \frac{b^2-4ac}{4a} \text{ or } t^2 = \frac{b^2-4ac}{4a^2}$$

$$\Rightarrow \qquad t = \frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \qquad x - d = \frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \qquad x - \left(\frac{-b}{2a}\right) = \frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$\text{and so} \qquad x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

A funny Quadratic Equation

P. Raghavendra, 3rd Street, NGGO Colony, Coimbatore.

During one of our visits to the library, I and my friend came across quite an interesting question in the chapter on Quadratic equations.

"Is it possible to find a quadratic equation $x^2 + mx + n = 0$ such that m, n are its two roots"?

Our initial attempt was as follows: Since m is a root of the equation, it will satisfy the equation and hence we get $m^2 + m(m) + n = 0$ or

$$m^2 + m^2 + n = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Similarly since n is also a root,

$$n^2 + mn + n = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Factorizing (2), $n(n+m)+n = 0$ or $n(n+m+1) = 0$ giving $n=0$ or $n = -(m+1)$. If we take $n = 0$ in (1), then we get $2m^2 + n = 0$ and hence $m= 0$.

If we take $n = -(m+1)$ in (1), then we get

$$m^2 + m^2 - (m+1) = 0 \;\Rightarrow\; 2m^2 - m - 1 = 0 \;\Rightarrow\; (2m+1)(m-1) = 0 \textit{ and so } m = -\frac{1}{2} \textit{ or } 1.$$

$$m = -\frac{1}{2} \textit{ gives } n = -(m+1) = -\frac{1}{2}; \quad m = 1 \textit{ gives } n = -(1+1) = -2$$

When we verify these solutions of the given equation we stumbled upon an obstacle. The solution $m = n = -\tfrac{1}{2}$ does not satisfy the given problem. We could not find the reason. So we worked for some alternate methods.

Given: $x^2 + mx + n = 0.$

$\therefore$ Sum of the roots $\quad m + n = -m \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$

Product of the roots $\quad mn = n \ldots\ldots\ldots\ldots\ldots\ldots(4)$

Equation (4) shows that $m = 1$ or $n = 0$.

If $m = 1$, then $n = -2$ from equation (3).

If $n = 0$, then $m = 0$ from equation (4).

The result $m = n = 0$ is notappealing; it is trivial. Using the other values the required equations is $x^2+x-2 = 0$.

Mathematicians on Page 1.

1. ***Srinivasa Ramanujan*** FRS(1887-1920) was an autodidact who, with almost no formal training in pure mathematics, made extraordinary contributions to analysis, number theory, infinite series, and continued fractions. (Born at Erode, Tamilnadu).
2. Born in 1920 at Hadagali in Karnataka, ***C R Rao***, FRS is a great name from the golden age of statistics. The American Statistical Association has described him as "a living legend whose work has influenced not just statistics, but has had far reaching implications for fields as varied as economics, genetics, anthropology, geology, national planning, demography, biometry, and medicine."
3. ***D R Kaprekar***, (1905-1986) born in Dahanu, a town about 100 km north of Mumbai, discovered several results in number theory, including a class of numbers and a constant named after him. Without any formal mathematical education he published extensively and was very well known in recreational mathematics circles.
4. Born in Kolkatta, Padma Vibhushan awardee, ***SatyendraNath Bose***, FRS (1894-1974) was an Indian physicist specialising in theoretical physics, best known for his work on quantum mechanics in the early 1920s, providing the foundation for Bose–Einstein statistics and the theory of the Bose–Einstein condensate.
5. Born (1957) in Gwalior, ***Narendra Krishna Karmarkar*** invented the first provably polynomial time algorithm for linear programming, also known as the interior point method. The algorithm is a cornerstone in the field of Linear Programming.
6. ***Srinivasa S. R. Varadhan*** was (born 1940) in Chennai. Currently Professor of Mathematics and Frank J. Gould Professor of Science at the Courant Institute of Mathematical Sciences, New York University, he is known for his fundamental contributions to probability theory.
7. ***Mahan Mj*** (born 1968), also known as Mahan Maharaj and Swami Vidyanathananda, is a monk of the Ramakrishna Order. Being a recipient of the 2011 S S Bhatnagar Award in Mathematical Sciences, he is currently Professor of Mathematics at the TIFR in Mumbai and best known for his work in hyperbolic geometry, geometric group theory, low-dimensional topology and complex geometry.
8. Born in 1973 in Chennai, ***Kannan Soundarrajan*** represented India at the International Mathl Olympiad in 1991 and won a Silver Medal. Currently a Professor of Mathematics and the Director of the Mathematics Research Center (MRC) at Stanford university, he.and his colleague Robert Lemke Oliver from Stanford University, USA, have found a strange pattern in prime numbers; that is they are not distributed as randomly as it was assumed.
9. ***R C Gupta*** [born 1935 in Jhansi(U.P)] is an Indian historian of mathematics He addressed with the history of mathematics, especially that of Indian trigonometry. In 1991 he was elected a Fellow of the National Academy of Sciences, India, and in 1994 he became President of the Association of Mathematics Teachers of India. In 1979 he founded the magazine <u>*Ganita Bharati*</u>. In 2009 he was awarded the Kenneth O. May Prize alongside the British mathematicianIvor Grattan-Guinness. He is the notably the first Indian to get this prize.

MAGIC CROSS - 2018

				25	844	633	516				
				750	399	142	727				
				376	493	984	165				
				867	282	259	610				
31	850	627	510	49	868	609	492	85	904	573	456
744	393	148	733	726	375	166	751	690	339	202	787
382	499	978	159	400	517	960	141	436	553	924	105
861	276	265	616	843	258	283	634	807	222	319	670
				81	900	577	460				
				694	343	198	783				
				432	549	928	109				
				811	226	315	666				
				91	910	567	450				
				684	333	208	793				
				442	559	918	99				
				801	216	325	676				

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Sample questions from AMTI Summer Camp (2018) for Sub-juniors.

01. You have two triangles, which altogether have six angles. Five of these angles are 50°, 60°, 70°, 80°, and 90°. How large (in degrees) is the sixth angle?

02. Two prime numbers P and Q have the property that both their sum and their difference are again prime numbers. What are P and Q?

03. On each day that Achyut does his homework his mother gives him Rs 4, and on days he 6 doesn't she takes Re 1 away from him. After 30 days Achyut notices that he has the same amount of money as when he started even though he has spent nothing and had no other source of income. On how many of the 30 days did he do his homework?

04. A prime number plus a perfect square equals 99. What is the prime number?

05. A quadrilateral has three sides of lengths 5.5, 6.5 and 7.5 metres. The length of the fourth side in metres is a positive integer. How many possible lengths (in metres) can the fourth side have?

06. If the 8-digit natural number 37A062BC is divisible by 720, find A.

07. What is the 2018th letter of the sequence
I L I K E M A T H I L I K E M A T H I L I K E M A T H...?

08. The date August 28, 2016 has the property that when this date is written in the format MMDDYYYY, all eight digits are even, i.e. 08282016. What is the next date after this one with this same property?

09. Find the largest positive integer X such that $\frac{2}{7}$ is smaller than $\frac{7}{X}$.

10. Ranjit was reading a book and was counting the number of 1s that appeared in the page numbers. He counted that there were 24 ones. If the book starts on page 1, how many pages does the book contain?

Answers for Puzzles:

Cross the bridge!

People crossing	*Time taken*	*Total time*
A and B cross	2	2
A returns	1	3
C and D cross	10	15
B returns	2	15
A and B cross	2	17

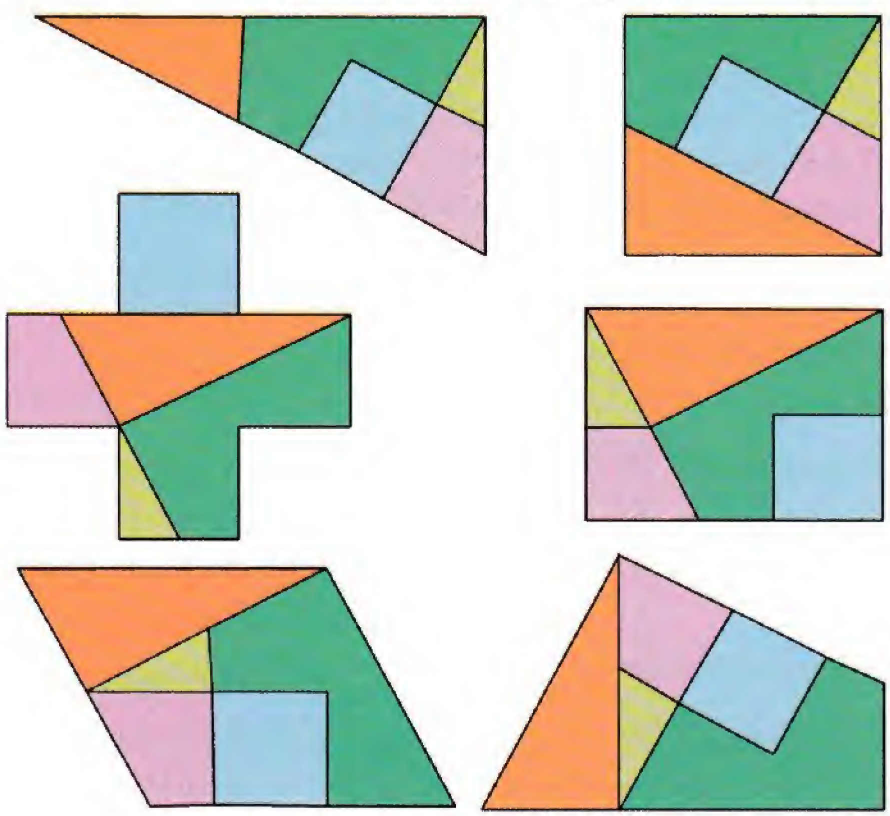
How do you generalize?

$$0+1+2+\cdots+n = \frac{(n+1)^2-(n+1)}{2} = \frac{(n+1)(n+1-1)}{2} = \frac{(n+1)n}{2}$$

A fallacy!

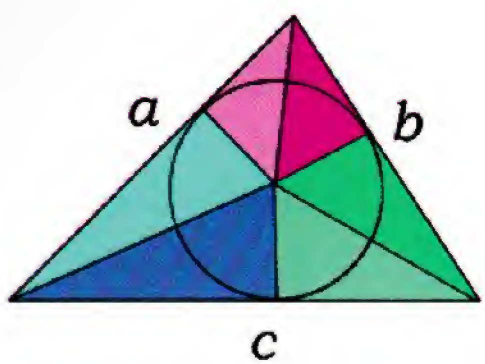
Next to step (1) you should have $\pm(n+1) = (n+2)$. The statement $+n+1 = n+2$ is absurd; the second statement $-(n+1) = n+2$ is correct and leads to $n = -1\frac{1}{2}$, which matches with our original assumption.

Making a Geometric figure!

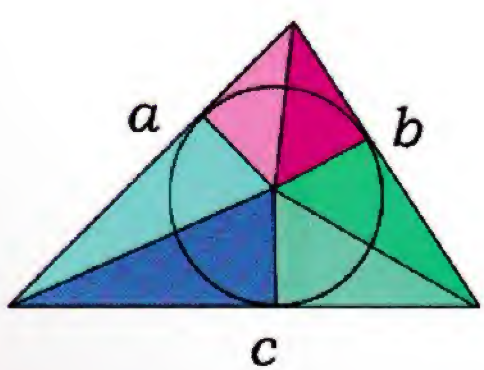


A Picture Story

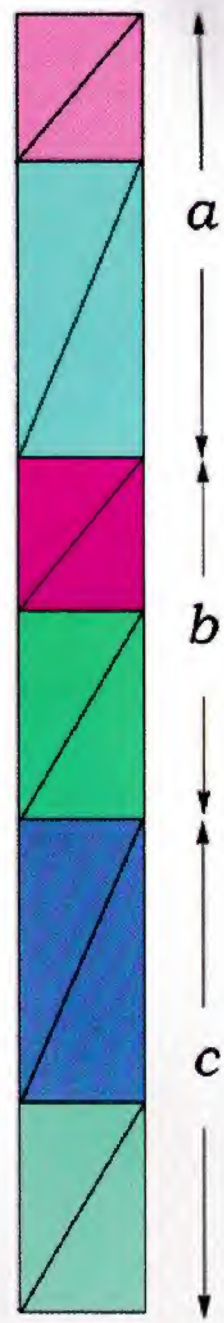
The Product of the Perimeter of a Triangle and its Inradius is Twice the Area of the Triangle



and



same as



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Instituted by Prof. M.S.Rangachari, a senior member of the AMTI and Retired Professor and Director, Ramanujan Institute for Advanced Study in Mathematics, University of Madras, a fund named PKS-JM Award Fund has been created by the AMTI. It is to award prizes to good articles of STUDENTS published in the Junior Mathematician every year from the year 2007 – 08.

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"CYCLIC NUMBERS"
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For article by
ii. Study Group X,
Sri Sarada Secondary School, Chennai - 600004.
"USING GRAPHS TO CREATE DESIGNS"
Vol 26-3 pages 33-38.

M.O. in their names is being sent. Congratulations to the winners. We hope that this will motivate prospective contributors to the JM.

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Printed Matter